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# LOW-THRUST ORBIT RAISING IN CONTINUOUS SUNLIGHT

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## SUMMARY

The altitude to which an Earth satellite can be raised and remain in continuous sunlight has been computed by an optimization of the initial orbit parameters and by a utilization of the oblateness of the Earth as the orbital plane rotation mechanism. Altitude raising is accomplished by a continuous tangential or circumferential thrust acceleration for thrust-weight ratios between $0.75 \times 10^{-5}$ and $2.00 \times 10^{-5}$. The computed missions start from circular orbits at an altitude of 300 or 500 nautical miles. Launch variable tolerances are discussed, and tables listing the optimum results are presented together with the methods of solution.

## INTRODUCTION

An Earth satellite that has a gradual spiral-out trajectory may be desirable for several types of missions. A spiral-out mission could be useful in making a thorough survey of the region of space near Earth. Such a survey could include, for example, a magnetic-field survey, a radiation-belt survey, or a survey of micrometeoroid densities at various distances from the Earth. In addition, such missions, which utilize a small, continuous constant thrust, are well oriented to testing of electric-propulsion thrust devices. These devices typically produce accelerations of the order of $10^{-5}$ to $10^{-4}$ times the acceleration of gravity over long periods of time with very low fuel expenditure.

Missions of this type would probably require continuous electrical power for continuous operation of the electric devices. Solar cells are, at present, the most feasible source of lightweight, long-duration electrical power for satellites. Since solar cells require continuous illumination for continuous power output, a trajectory that remains in continuous sunlight is necessary.

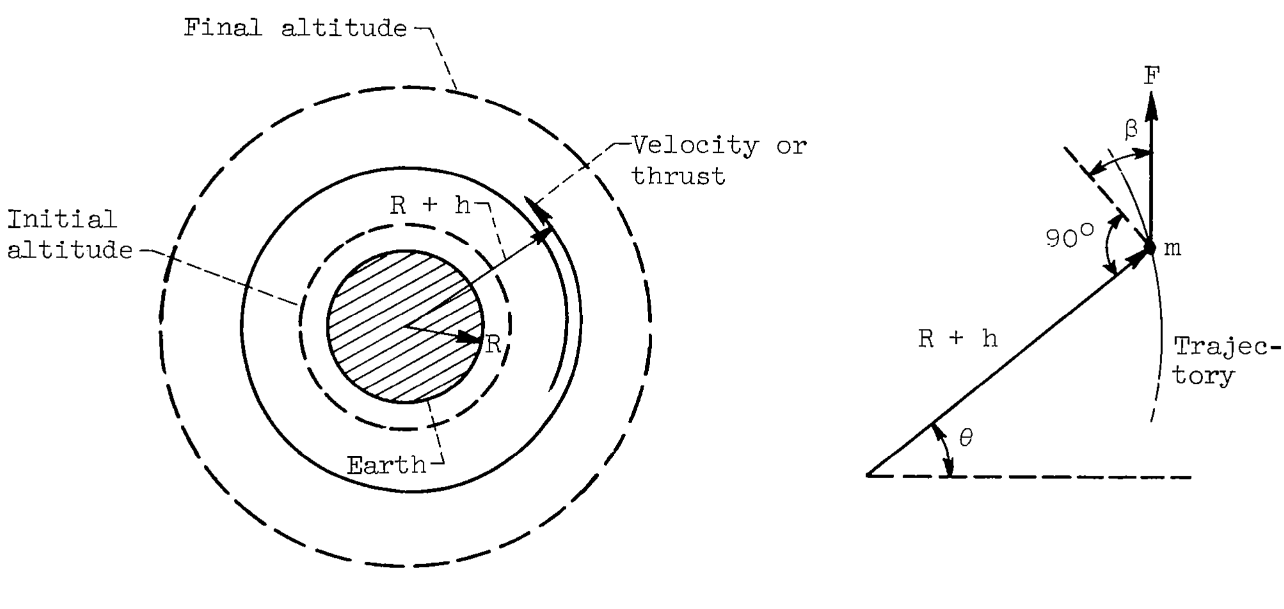
For an orbit to remain in continuous sunlight it must be oriented so that the sun remains near the perpendicular to the orbit plane. As the sun appears to move along the ecliptic during the year, the plane of the orbit must be rotated in order to maintain the sun in the desired vicinity of the perpendicular. It is known that the oblateness of the Earth causes a precession of the plane of an orbit about the Earth's polar axis. The use of this oblateness precession to rotate the orbit plane by the amount needed to keep a satellite in continuous sunlight has been treated by other authors (refs. 1 and 2). Previous studies, however, have not established the optimum conditions for maximizing the altitude or mission time that can be attained.

This study has been made to compute the maximum altitude and, consequently, the mission time that an Earth satellite can be expected to attain under the action of a continuous, constant tangential or circumferential thrust acceleration, while remaining in continuous sunlight. It will be shown that the initial orbit parameters and their tolerances, as well as the thrust acceleration, affect the maximum altitude and mission time. These orbit parameters consist of orbital inclination, orbital altitude, and position of orbit node and orbit perpendicular with respect to the sun. With thrust being either tangential or circumferential and thrust-weight ratios between $0.75\times10^{-5}$ and $2.00\times10^{-5}$, initial orbit parameters have been optimized to give the maximum altitude and mission duration. For simplicity, initial orbit altitudes of 300 and 500 nautical miles have been considered.

## ANALYSIS

### Spiral-Out Trajectories

The instantaneous altitude of a satellite in some arbitrary orbit will be increased by the application of a continuous thrust acceleration, either circumferential or tangential to the trajectory. Reference 3 has shown that, if the starting orbit is circular, the trajectories for the thrust-weight ratios considered in this report will remain nearly circular at least up to 20,000 nautical miles or for mission durations up to 1 year. In other words, the gradually increasing spiral can be approximated everywhere by a circular orbit. This spiral is shown greatly exaggerated in sketch (a). At the instant depicted, the spiral can be approximated by a circular orbit of radius $R + h$. (Symbols are defined in appendix A.)




(a) (b)

Shown in sketch (b) is a satellite of mass $m$ acted upon by a thrust $F$ in the presence of a central gravitational field having a gravitational constant $\mu$.

The equations of motion can be written as follows:

$$\left.\begin{aligned}\ddot{h} - (R + h)\dot{\theta}^2 &= -\frac{\mu}{(R + h)^2} + \frac{F}{m}\sin\beta \\ 2\dot{h}\dot{\theta} + (R + h)\ddot{\theta} &= \frac{F}{m}\cos\beta = ag\cos\beta\end{aligned}\right\}\qquad(1)$$

where $a$ is the thrust-weight ratio the satellite would have if it were on the equator at the Earth's surface. The acceleration of gravity $g$ is also associated with the equator at the Earth's surface. The altitude $h$ is measured above the equatorial radius $R$.

If the assumption is made that the trajectory remains everywhere circular, which means that the velocity on the trajectory can be expressed by $(R + h)\dot{\theta}$ or $\sqrt{\mu/(R + h)}$, equations (1) can be reduced to

$$\dot{h} = \frac{2\sqrt{g}}{R} a(R + h)^{3/2} \qquad (2)$$

Inserting the appropriate constants to give $R$ and $h$ in nautical miles and time in days and integrating give the time $\Delta t$ required to increase the altitude from $h_0$ to $h$ as

$$\Delta t = \frac{0.549}{a}\left[(R + h_0)^{-1/2} - (R + h)^{-1/2}\right] \qquad (3)$$

## Earth Oblateness Effect

As indicated in figure 1, the oblateness of the Earth gives rise to a torque on the orbit about an axis through the equatorial nodes. The attraction by the excess equatorial mass is in a direction to change the inclination of the orbit, but, because of the gyroscopic nature of the orbit, the inclination does not change significantly (ref. 4). Instead, the orbit precesses about the polar axis of the Earth, and a change $\Delta\Omega$ in the location of the orbit equatorial nodes results. The position of the orbital node $\Omega$ is measured positively eastward from the vernal equinox. The angle $\Delta\Omega$ is also the change in the angle that the orbit perpendicular makes with the direction of the vernal equinox when projected on the equatorial plane. In the special cases of inclinations of $0^\circ$, $90^\circ$, or $180^\circ$, oblateness causes no torque on the orbit, and there is no resultant precession. For the purpose of this report, the inclination $i$ is measured at the ascending node, $0^\circ$ to $180^\circ$, from the easterly direction counterclockwise to the orbit track.

An analytical expression for the rate of precession of nodes of an orbit can be obtained by considering only the terms through the second harmonic in the potential function of the oblate Earth. The following expression is obtained in reference 4 by ignoring the smaller terms in the potential function:

$$\dot{\Omega} \approx -\sqrt{\mu}\, JR(R + h)^{-7/2}\cos i \qquad (4)$$

where $J$ is the coefficient of the second harmonic in the gravitational potential function. Note that for inclinations greater than 90° (retrograde orbits) $\dot{\Omega}$ is positive, which results in the eastward motion of the node. When the proper values are given to the constants $\mu$, $J$, and $R$, equation (4) yields

$$\dot{\Omega} = -9.96\left(\frac{R}{R + h}\right)^{7/2} \cos i \tag{5}$$

with $\dot{\Omega}$ in degrees per day.

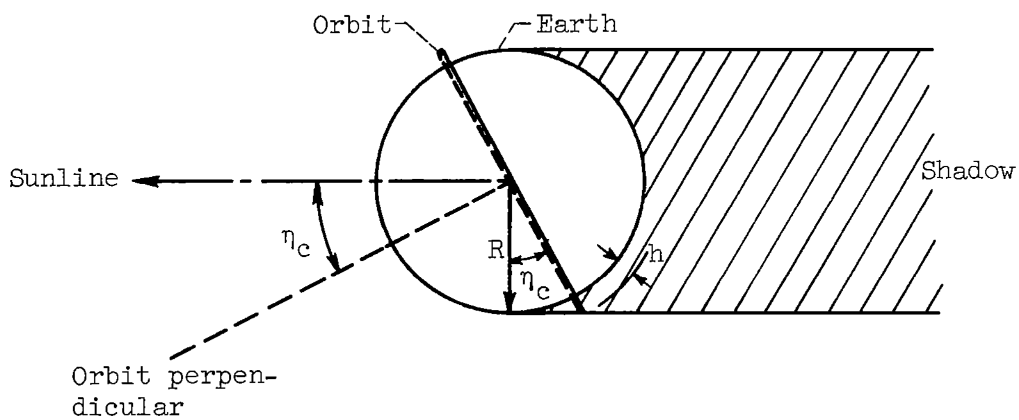
When equations (5) and (2) are combined and integrated, the change in the location of the orbit nodes or the orbit perpendicular can be found as a function of the thrust-weight ratio and the instantaneous altitude. Thus,

$$\Delta\Omega = -1.637\times10^{12} \frac{\cos i}{a} \left[(R + h_0)^{-4} - (R + h)^{-4}\right] \tag{6}$$

where $\Delta\Omega$ is in degrees when $R$ and $h$ are in nautical miles.

## Earth Shadow Considerations

By assuming only circular orbits, the geometry of the orbit with respect to illumination from the sun is simplified. It can be seen from sketch (c) that, as long as angle $\eta$ between the orbit perpendicular and the sunline is less than



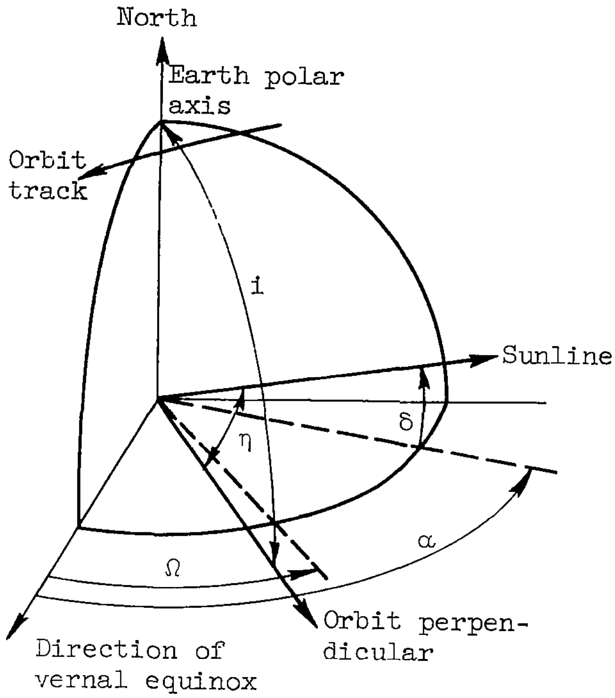

Plane of sketch is plane containing sunline and orbit perpendicular.

(c)

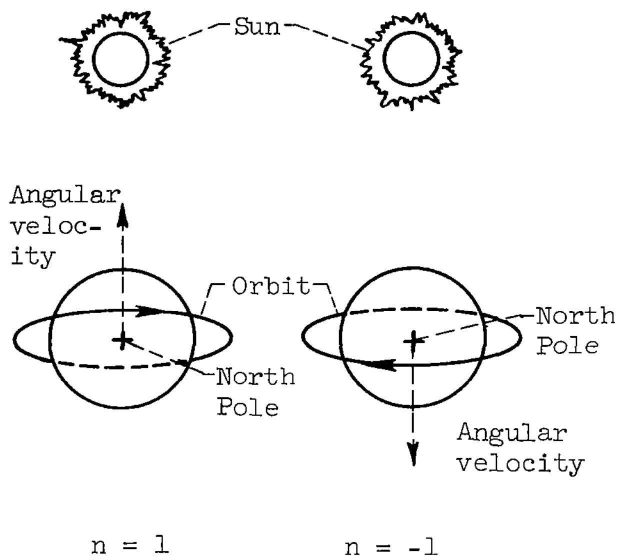
$\eta_c$, the orbit will be in continuous illumination. Consequently, $\eta_c$ is defined by

$$\cos \eta_c = \frac{R}{R + h} \qquad (0 < \eta_c < 90°) \tag{7}$$

This means that for an orbit to be in continuous sunlight the orbit perpendicular must remain inside a cone having the moving sunline as its axis. The half-angle of this cone is the $\eta_c$ corresponding to the instantaneous orbital altitude. This cone, shown in figure 2, is called the cone of tolerance.




(d)



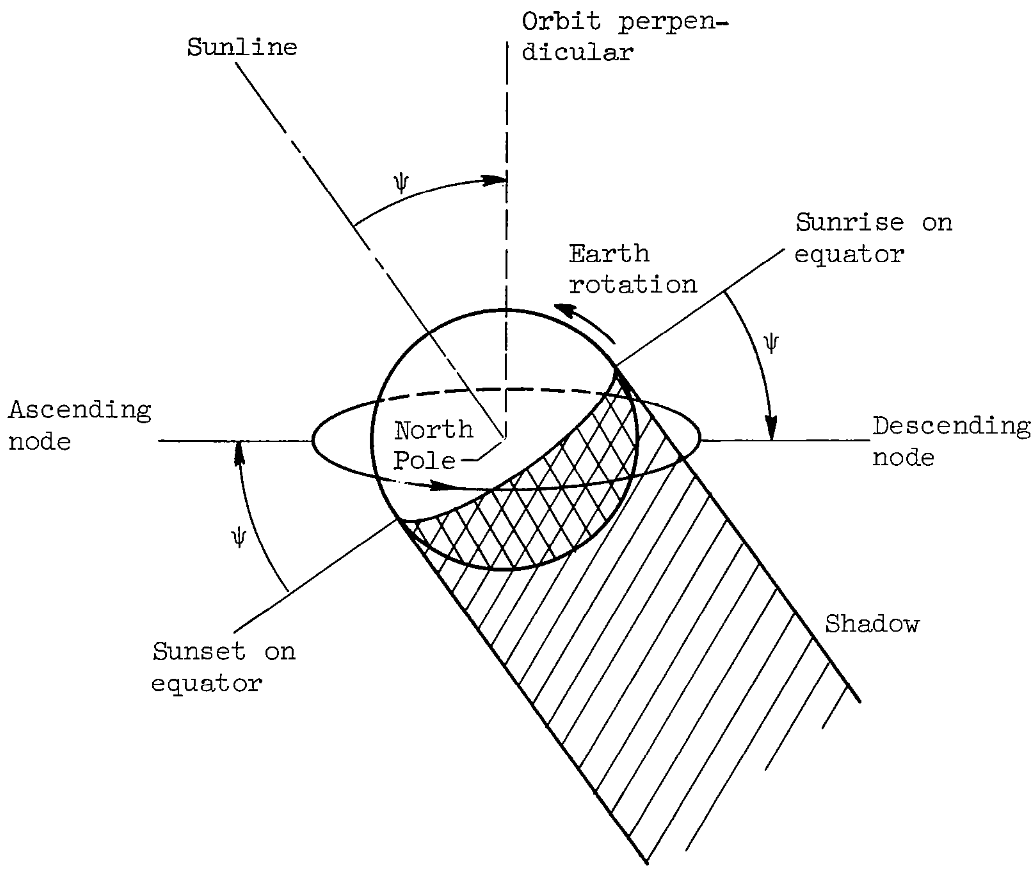

(e)

An expression for the angle $\eta$ can be derived from sketch (d). The orbit perpendicular and the sunline are considered to be vectors in a nonrotating Earth-centered reference system. The vector dot product between the two vectors gives

$$\cos \eta = \cos \delta \sin i \cos(\alpha - \Omega) + n \sin \delta \cos i \tag{8}$$

where the constant $n$ is either 1 or -1, depending on whether the sun is on the same side of the orbit as the angular velocity vector or on the opposite side (sketch (e)).

The angle $\psi$ is defined to be the difference between $\alpha$ and $\Omega$. As such it appears in equation (8) and is an indication of how far the orbit perpendicular is west of (lagging behind) the easterly moving sunline. From sketch (f) it can be seen that $\psi$ is also the angular distance before sunrise or sunset at which the orbit crosses the equator. If the apparent solar time of sunrise or sunset on the equator is always 0600 and 1800, $\psi$ can be interpreted as time. In fact, the apparent solar time under the orbit at any latitude can be computed if $\psi$, i, n, and the latitude in question are known. Equations for apparent solar time are derived in appendix B as a means of investigating requirements of




(f)

certain launch parameters.

## Dynamics of Problem

With the annual motion of the Earth about the sun, $\alpha$ and $\delta$ change continuously. While $\alpha$ is an increasing function, $\delta$ varies between approximately $23\frac{1}{2}^{\circ}$ and $-23\frac{1}{2}^{\circ}$. Since the sunline is the axis of the cone of tolerance, changing $\alpha$ and $\delta$ moves the cone of tolerance along the ecliptic as in figure 3. Each location of the axis of the cone corresponds to a specific date.

As time progresses, the altitude of the orbit or trajectory increases according to equation (3). This results in an increasing $\eta_c$, or half-angle, of the cone of tolerance as it moves along the ecliptic. In addition, the increase in altitude causes a decreasing rate of change of $\Omega$ with time, according to equation (5). Hence, some relative motion between the orbit perpendicular and the sunline exists.

From figure 2 it can be seen that this relative motion of the orbit perpendicular depends on starting conditions. The mission is assumed to commence with the initiation of thrust. Mission starting date determines the initial right ascension and declination of the sun, $\alpha_0$ and $\delta_0$. Initial altitude determines, with inclination, the initial $\dot{\Omega}$. Initial altitude also determines $\eta_{c,0}$. The starting position of the orbit perpendicular is determined by $\psi_0$ and i. A limit is placed on $\psi_0$ and i by the initial altitude because the values must fall inside the circle of tolerance defined by the intersection of the cone of tolerance and the surface of the Earth.

If, for simplicity, it is assumed that the circle of tolerance does not expand with time, figure 4 shows the motion of the orbit perpendicular for typical starting conditions that differ only in $\psi_0$. For cases 1 to 6 the inclination is such that the initial precession of the orbit is greater than the initial increase in $\alpha$. As $\psi_0$ is increased from case 1 to 6, the length of the path followed by the orbit perpendicular increases for curves 1, 2, and 3. This is equivalent to increasing the mission time and thus increasing the final altitude. The mission terminates when $\eta = \eta_c$ or when the orbit perpendicular passes outside the circle of tolerance.

When $\psi_0$ is increased to curve 4, there is a step increase in the mission time as the orbit perpendicular just approaches the edge of the circle of tolerance and then drifts back to the other side. This results in two discontinuous sets of functions for altitude. Increasing $\psi_0$ further beyond curve 4 decreases the altitude that can be attained.

The maximum altitude possible for a certain mission starting date and spiral-out rate will occur when the inclination is such that, with a $\psi_0$ as large as possible, the orbit perpendicular will just approach the limit on the opposite side. This is illustrated in figure 5, where the expansion of the circle of tolerance is shown more as it actually would be. The shape and, consequently, the length of time that elapses between points 1 and 3 on the path of the orbit perpendicular depends on nearly all the initial orbit parameters mentioned thus far. There is, however, one set of parameters that will provide the maximum elapsed time, and this is the solution desired.

A final consideration is the effect of error tolerances on the initial orbit parameters due to launch inaccuracies. These tolerances, especially on i, $\psi_0$, and $h_0$, will limit how close the orbit perpendicular should be programmed to approach the circle of tolerance as in figure 4. If insufficient tolerance is allowed, the orbit may enter shadow on the close approach to the edge of the circle of tolerance, or it may even be in shadow before the start of the mission. Attaining the maximum mission time and altitude, as illustrated in figure 5, would require precise control of the orbit parameters.

## RESULTS AND DISCUSSION

In general, two methods of approach were used to obtain the optimum conditions for maximum altitude. Computations were carried out on an IBM 7094 computer because of the large number of variables and the wide range of each that had to be analyzed.

The first method of approach limited the number of variables to be optimized by first making the assumption that the maximum altitude would be attained when the orbit perpendicular initially started as far behind the sunline as possible. Thus, $\psi_0$ was determined by

$$\cos \eta_{c,0} = \cos \delta_0 \sin i \cos \psi_0 + n \sin \delta_0 \cos i \tag{9}$$

Right ascensions and declinations of the sun were obtained from ephemeris tables as functions of date. The angle $\psi_0$ is a function of initial altitude, inclination, and mission starting date.

For a specific starting date each inclination was checked to see how long the mission could proceed before the orbit entered shadow. This involved a day by day comparison of $\eta$ and $\eta_c$ as computed from equations (3), (6), (7), (8), and (9).

Figure 6 shows altitude as a function of time from equation (3), and figure 7 is a plot of equation (7). The inclination that gave the longest mission time and the highest altitude before entering shadow was then selected as the optimum for that date. This process was repeated with a change in $n$ to obtain a second set of data. Advancing the starting date and repeating again gave a yearly variation of maximum altitude and optimum initial orbit conditions for different starting dates.

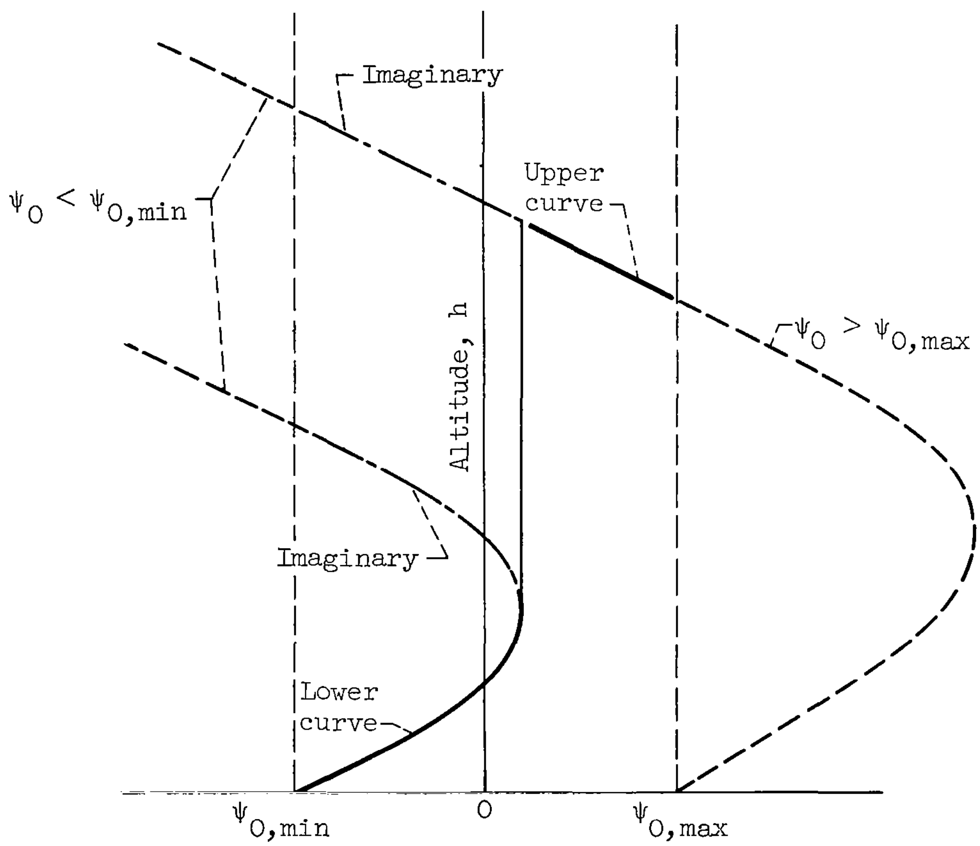
The entire program was run for initial altitudes of 300 and 500 nautical miles and thrust-weight ratios of $0.75\times10^{-5}$, $1.00\times10^{-5}$, $1.25\times10^{-5}$, $1.50\times10^{-5}$, and $2.00\times10^{-5}$. The results are plotted in figure 8, and the maximum points on each curve are tabulated together with the corresponding $i$ and $\psi_0$ in table I. If the altitudes specified by these conditions are to be reached, there can be no error tolerance allowed. These values of altitude are essentially absolute maximums. A typical time history of $\eta$ and $\eta_c$ is shown in figure 9 for $a = 1.25\times10^{-5}$ and $h_0 = 300$ nautical miles. The orbit perpendicular approaches the edge but does not leave the circle of tolerance at a time of about 100 days.

Since realistic satellite launchings always have some tolerance on inclination, $\psi_0$, and initial altitude, a second method of selecting optimum variables was devised. Using a simultaneous solution of the equations that were used in the first program, with the exception of equation (9), and setting $\eta = \eta_c$ yield

$$R\left[(R + h_0)^{-1/2} - \frac{a\,\Delta t}{P}\right]^2 = \cos \delta \sin i \cos\left(\alpha - \alpha_0 + \psi_0 + \frac{Q \cos i}{a} \times \left\{(R + h_0)^{-4} - \left[(R + h_0)^{-1/2} - \frac{a\,\Delta t}{P}\right]^8\right\}\right) + n \sin \delta \cos i \tag{10}$$

where $P = 0.549$ and $Q = 1.637\times10^{12}$. This equation gives the mission duration $\Delta t$ that can be expected with a particular $a$, $h_0$, $i$, $\psi_0$, $n$, and mission starting date. The starting date gives $\alpha_0$ directly, and $\alpha$ and $\delta$ are tabular functions of the starting date plus $\Delta t$.

It was decided not to attempt to solve equation (10) directly for $\Delta t$. Instead, it is much easier to assume a $\Delta t$, i, starting date, a, and $h_0$ and to then solve for $\psi_0$. This is equivalent to assuming a solution and solving for the starting conditions. Incrementing $\Delta t$ over some range and computing the altitude associated with each $\Delta t$ result in families of curves of altitude as a function of i and $\psi_0$. As might be expected, such a procedure results in some starting conditions that are physically impossible and some that lead to $\Delta t$'s that are imaginary. Sketch (g) indicates how a typical curve for one inclination




(g)

and one mission starting date might look. It has two separate areas - an upper and a lower curve. The two areas are what would be suggested by figure 4. The break between the lower and the upper areas occurs between paths 3 and 4.

Legitimate values from this second program are plotted in figure 10. The mission starting dates selected were the ones that gave the absolute maximum altitudes in table I from the first program. As shown in figure 10(c-1), for some inclinations all values of $\psi_0$ result in altitudes on the lower curve. This condition plus the maximum $\psi_0$ possible establishes a boundary on the upper family of curves. Beyond the dashed limits on the upper family, values exist in the lower family or not at all. For simplicity the lower family of curves has been omitted from the remaining figures.

The absolute maximum from the first program is at the corner of the boundary on the upper family of curves. This case is quite sensitive to $i$ and $\psi_0$. Small variations could place the point beyond either boundary. With a launch vehicle, such as the Agena, it may be possible to establish the desired orbital parameters with a high degree of accuracy. If this cannot be done, a certain tolerance on these variables must be accepted. The desired orbital parameters that will give the highest probability of achieving a reasonably high altitude considering the variations can be found from figure 10.

If in figure 10(c-1), for example, a trapezoid is drawn whose sides are equal to twice the error expected in $\psi_0$ and $i$, the result is the altitudes possible when aiming for the center of the trapezoid. When the location of this trapezoidal area is adjusted to the highest altitude, with the area still inside the boundaries of the upper family of curves, the center is the optimum aim point, and the range of possible altitudes is defined by the extremities. If there is a tolerance on $h_0$, it can be illustrated by using three plots, such as figures 10(c-1) and 11(a) and (b), with $h_0$ equal to the nominal altitude and the nominal altitude plus and minus the tolerance. The error trapezoid must remain within the boundaries for all three curves.

The results of error considerations are listed in table II for $\pm 1^\circ$ tolerance on $i$ and $\psi_0$ for all cases in figure 10 and for $h_0 = 300$ nautical miles $\pm 10$ percent in figures 10(c-1) and 11.

Figure 12 (for $i$ and $\psi_0$ error tolerances of $\pm 1^\circ$) can be compared with figure 9 (the ideal case) to illustrate the effect on $\eta$ of selecting the revised aim point based on error tolerances. Figures 9 and 12 can also be used to determine the orientation that solar panels might need on a mission of this sort.

## CONCLUDING REMARKS

It has been shown that the altitude, or mission time, that can be attained in continuous sunlight can be maximized by a choice of optimum starting conditions. These conditions have been determined for several thrust-weight ratios and initial altitudes. To some extent the degree of difficulty in establishing the desired initial orbit conditions and the effect of inaccuracies in these conditions have been investigated.

Some interesting conclusions can be drawn from the results of tables I and II. First, it appears that the maximum altitudes occur with launch dates in autumn for the range of thrust-weight ratios and initial altitudes considered. The gain in autumn launches over spring launches ranges from 300 to 1000 nautical miles and 1 to 17 days in mission time. The greatest increase in altitude is for the highest thrust-weight ratio, while the largest increase in mission time is for the lowest thrust-weight ratio.

Another conclusion is that the inclinations for the optimum starting conditions are greater than $90^{\circ}$ and increase with increasing thrust-weight ratio and increasing initial altitude. In other words, the orbits are retrograde.

From figures 10 and 11 it can be seen that the final altitude is very sensitive to variations in inclination and initial altitude and is less sensitive to changes in $\psi_0$ (associated with time of day at launch).

Finally, a comparison of the values listed in table II shows, as might be expected, that the higher the initial altitude or the greater the thrust-weight ratio the higher will be the altitude attained. Mission duration, on the other hand, responds somewhat differently. Over the range investigated, the smaller the thrust-weight ratio, the longer the mission time. For an electric-propulsion life test, longer mission time and smaller thrust-weight ratio might be desirable. There does not seem to be too great a difference in mission time between initial altitudes of 300 and 500 nautical miles for any one thrust-weight ratio.

It appears feasible that a mission could be planned around the concept of continuous sunlight with increasing orbital altitude by using the approach described in this report. A major problem would be the accurate establishment of the initial orbit. Another difficulty might be long-term cumulative effects from unaccounted for higher order oblateness terms. As refinements along these lines become apparent, they can be added to the general method of approach outlined in this report to provide more accurate results.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, November 12, 1963

## APPENDIX A

## SYMBOLS

| | |
|---|---|
| $a$ | thrust-weight ratio, $F/mg$ |
| $F$ | thrust |
| $g$ | gravitational acceleration on equator at Earth's surface, 9.780320 m/sec$^2$ |
| $h$ | altitude of satellite above equatorial radius, international nautical miles |
| $i$ | inclination of orbit or trajectory plane |
| $J$ | coefficient of second harmonic in gravitational potential function, $1623.42\times10^{-6}$ |
| $m$ | mass of satellite |
| $n$ | constant in eq. (8) 1 or -1, depending on orbit orientation |
| $R$ | equatorial radius of Earth, 3444.058 international nautical miles |
| $T$ | mission starting date, days after vernal equinox |
| $\Delta t$ | mission time, days |
| $\alpha$ | right ascension of sun |
| $\beta$ | angle between thrust vector and perpendicular to radius vector |
| $\gamma$ | angle, $\psi_0 + 90^\circ - \varphi$ (see sketch (h)) |
| $\delta$ | declination of sun |
| $\eta$ | angle between sunline and orbit perpendicular |
| $\eta_c$ | maximum $\eta$ for orbit in continuous sunlight |
| $\theta$ | polar angle on trajectory plane |
| $\lambda$ | latitude (northern hemisphere only) |
| $\mu$ | gravitational constant of Earth, $1.40772\times10^{16}$ cu ft/sec$^2$ |
| $\tau$ | apparent solar time |
| $\varphi$ | angle between meridian through launch site and orbit node (see sketch (h)) |
| $\psi$ | angle by which orbit perpendicular lags sun in longitude, $\alpha - \Omega$ |

$\Omega$ right ascension of orbit perpendicular, angle between vernal equinox and projection of orbit perpendicular on equatorial plane

$\Delta\Omega$ change of $\Omega$ in time $\Delta t$

Subscripts:

max maximum

min minimum

0 initial ($\Delta t = 0$)

Superscripts:

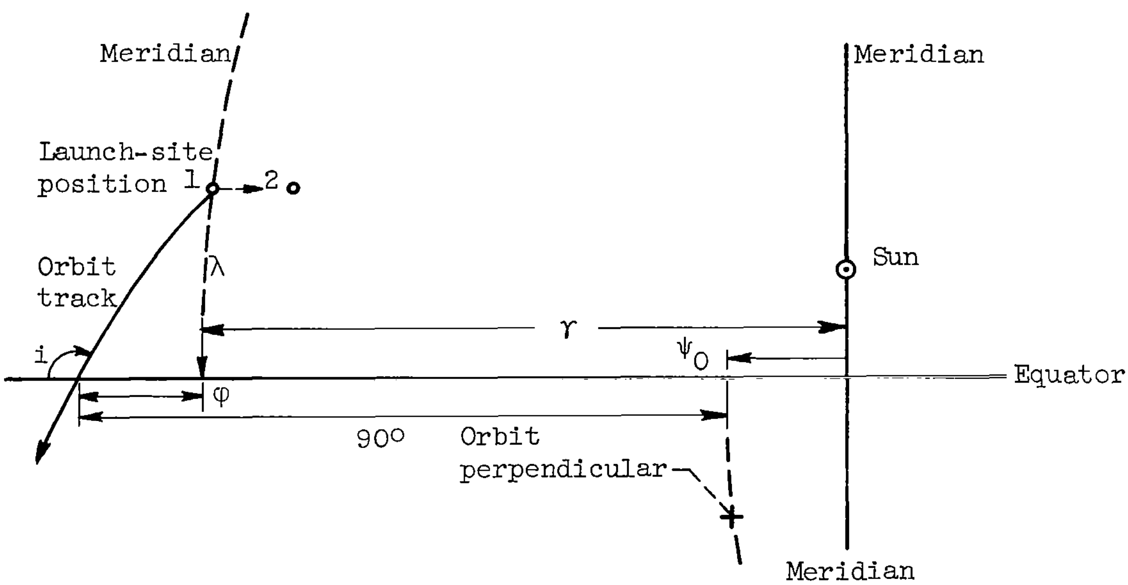
$\dot{}$ first derivative with respect to time

$\ddot{}$ second derivative with respect to time

## APPENDIX B

### APPARENT SOLAR TIME OF LAUNCH

If an orbit is to be oriented in space with a certain inclination and $\psi_O$, and if the launch site is farther from the North Pole in latitude than $|i - 90^\circ|$, the orbit can be established with a coplanar launch at only two times during a day. A coplanar launch is obtained when the final orbit and transfer trajectory are coplanar with the launch site at the time of launch. One of the two coplanar launch windows will require a launch azimuth in a northerly direction (up to $90^\circ$ to either side of true north), and the other window will require a southerly launch. Since the orientation of the orbit plane is established for coplanar launch, once the launch vehicle is outside the sensible atmosphere or, relatively speaking, just off the launch pad, the two launch windows will occur when the launch site is in the plane of the desired orbit. A coplanar launch at any other time would place the vehicle in a different orbital plane. Any plane rotation perturbations and the motion of the Earth about the sun having been neglected, if the conditions mentioned at the beginning of this paragraph are met, Earth rotation will cause the launch site to pass through the desired orbital plane twice a day.




(h)

From the geometry of sketch (h), the apparent solar time for the coplanar southerly launch window to establish an orbit with a specified $i$ and $\psi_O$ from a launch site of latitude $\lambda$ can be approximately determined. It should be noted that in this example $n = 1$ and $i > 90^\circ$. A similar approach could be made with any combination of northerly or southerly launches and values of $i$ and $n$. In this example it is assumed that position 1 is the location of the launch site at time $\tau$. At this particular instant it is possible to effect a coplanar launch into the desired orbit. Launch is unfavorable at a later time when the launch site has moved to position 2 because of the rotation of the Earth. The apparent solar time (at position 1) that the launch window occurs is given by the angular separation between the meridian through the launch site (at position 1) and the meridian through the sun $\gamma$. Since the time on the sun's meridian is always noon and $15^{\circ}$ of longitude is equivalent to 1 hour,

$$\tau \equiv 12.00 - \frac{\gamma}{15} \tag{B1}$$

where $\tau$ is the time in hours on a 24-hour clock. From sketch (h)

$$\gamma = \psi_0 + 90^{\circ} - \varphi \tag{B2}$$

Solving the spherical triangle indicated gives

$$\tan \lambda = \tan(180^{\circ} - i)\sin \varphi \tag{B3}$$

Since $\tan(180^{\circ} - i) = -\tan i$,

$$\gamma = 90^{\circ} + \psi_0 - \sin^{-1}\left(-\frac{\tan \lambda}{\tan i}\right) \tag{B4}$$

Substitution into equation (B1) gives

$$\tau = 12.00 - 6.00 - \frac{\psi_0}{15} + \frac{1}{15}\sin^{-1}\left(-\frac{\tan \lambda}{\tan i}\right) \tag{B5}$$

It can be found with a similar approach that when $n = -1$ only the second term (-6.00) changes sign. The resultant equation for $n = 1$ is

$$\tau = 12.00 - 6.00n - \frac{\psi_0}{15} + \frac{1}{15}\sin^{-1}\left(-\frac{\tan \lambda}{\tan i}\right) \tag{B6}$$

To convert $\tau$ to the usual 24-hour clock system, it is necessary to multiply the decimal parts of $\tau$ by 60 and to write in hours and minutes.

Equation (B6) can be used to determine the apparent solar time of a southerly launch (for a coplanar ascent trajectory) from a launch site of latitude $\lambda$ (northern latitudes only) into an orbit specified by $i$ and $\psi_0$. This has been done in table II for southerly launches from the Pacific Missile Range.

If $\psi_0$ is changed to $\psi$ in equation (B6), $\tau$ becomes the apparent solar time at a point having a latitude $\lambda$ when the southerly moving portion of the orbit having instantaneous $i$ and $\psi$ is directly overhead.

The apparent solar time mentioned in this report is time determined by direct reference to the actual position of the sun. Other times, such as the Greenwich mean time or the local time, can be determined by reference to any standard ephemeris.

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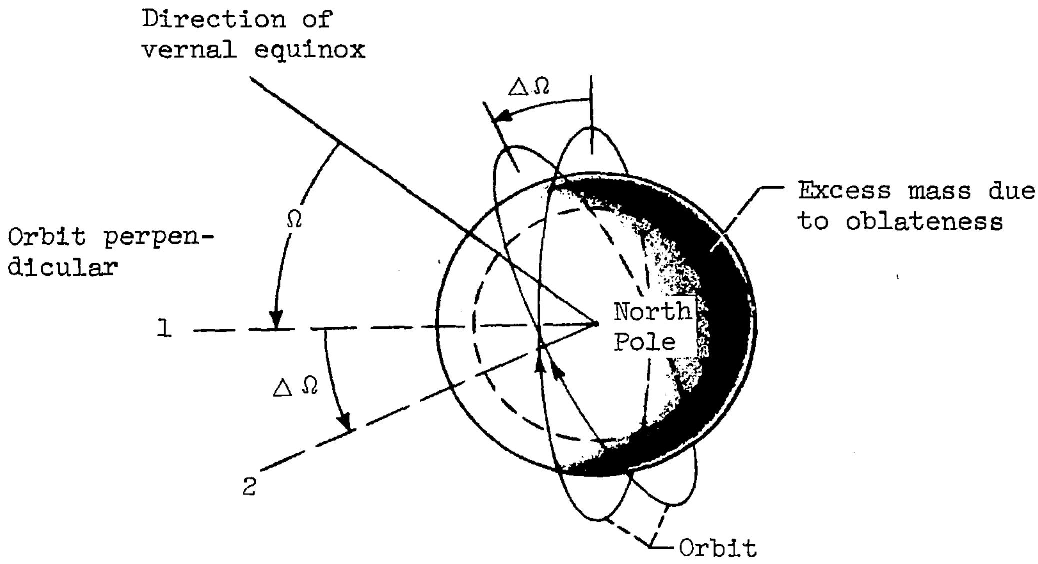
TABLE I. - ABSOLUTE MAXIMUM ALTITUDES WITH NO TOLERANCE ON LAUNCH VARIABLES

| Thrust-weight ratio, a | Initial altitude, $h_0$, nautical miles | Summer (constant n, -1) | | | | | Winter (constant n, 1) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Mission starting date | Maximum altitude, nautical miles | Mission time, days | Orbit inclination, i, deg | Initial angle by which orbit perpendicular lags sun in longitude, $\psi_0$, deg | Mission starting date | Maximum altitude, nautical miles | Mission time, days | Orbit inclination, i, deg | Initial angle by which orbit perpendicular lags sun in longitude, $\psi_0$, deg |
| $0.75\times10^{-5}$ | 300 | Apr. 20 | 3940 | 344 | 106.9 | 23.12 | Oct. 17 | 4240 | 361 | 107.4 | 22.14 |
| | 500 | Apr. 10 | 4740 | 356 | 111.5 | 26.66 | Oct. 12 | 4940 | 366 | 111.9 | 26.20 |
| $1.00\times10^{-5}$ | 300 | May 10 | 5180 | 306 | 109.3 | 24.[illegible] | Nov. 1 | 5610 | 320 | 110.0 | 23.40 |
| | 500 | Apr. 30 | 6130 | 313 | 114.9 | 2[illegible].07 | Nov. 1 | 6560 | 325 | 115.5 | 28.70 |
| $1.25\times10^{-5}$ | 300 | May 20 | 6630 | 280 | 111.6 | 24.64 | Nov. 21 | 7200 | 292 | 112.2 | 24.63 |
| | 500 | May 10 | 7750 | 284 | 118.2 | 29.54 | Nov. 11 | 8310 | 294 | 118.8 | 29.20 |
| $1.50\times10^{-5}$ | 300 | May 20 | 8420 | 262 | 114.0 | 24.52 | Dec. 1 | 9010 | 270 | 114.3 | 24.90 |
| | 500 | May 10 | 9580 | 262 | 121.3 | 28.35 | Nov. 21 | 10300 | 270 | 122.0 | 29.60 |
| $2.00\times10^{-5}$ | 300 | May 10 | 13600 | 238 | 118.2 | 22.25 | Dec. 6 | 14400 | 243 | 118.9 | 24.65 |
| | 500 | Apr. 20 | 15200 | 236 | 124.8 | 19.01 | Nov. 26 | 15400 | 237 | 128.1 | 27.26 |

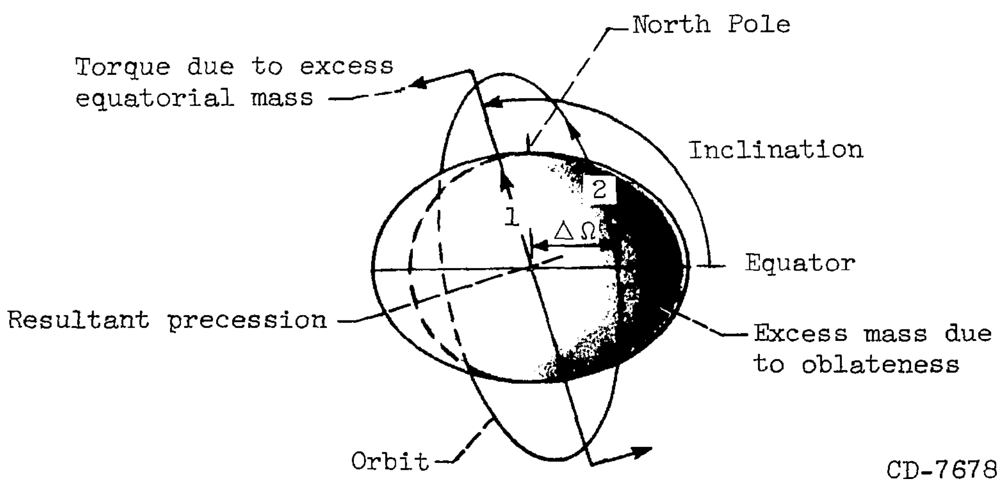
TABLE II. - SUMMARY OF MAXIMUM ALTITUDES WITH TOLERANCES OF $\pm 1^{\circ}$ ON $i$ AND $\psi_0$

| Thrust-weight ratio, a | Initial altitude, $h_0$, nautical miles | Constant, n | Optimum mission starting date | Aim point | | | Altitude reached, nautical miles | | | Mission time, days | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Orbit inclination, i, deg | Initial angle by which orbit perpendicular lags sun in longitude, $\psi_0$, deg | Launch time at Pacific Missile Range (a) | Aim | Maximum | Minimum | Aim | Maximum | Minimum |
| $0.75\times10^{-5}$ | 300 | 1 | Oct. 17 | 106.3 | 21.20 | 0524 | 3580 | 4230 | 2970 | 323 | 360 | 282 |
| | 500 | | Oct. 12 | 110.7 | 25.00 | 0520 | 4320 | 4940 | 3730 | 335 | 366 | 301 |
| $1.00\times10^{-5}$ | 300 | 1 | Nov. 1 | 108.8 | 22.25 | 0525 | 4800 | 5600 | 4150 | 292 | 320 | 267 |
| | 500 | | Nov. 1 | 114.3 | 27.50 | 0523 | 5800 | 6550 | 5070 | 303 | 325 | 279 |
| $1.25\times10^{-5}$ | 300 | 1 | Nov. 21 | 111.0 | 23.75 | 0527 | 6200 | 7150 | 5400 | 270 | 291 | 251 |
| | 500 | | Nov. 11 | 117.6 | 28.30 | 0531 | 7370 | 8260 | 6570 | 277 | 293 | 260 |
| $1.50\times10^{-5}$ | 300 | 1 | Dec. 1 | 113.1 | 24.00 | 0529 | 7850 | 8980 | 6900 | 254 | 270 | 238 |
| | 500 | | Nov. 21 | 120.6 | 28.70 | 0541 | 9130 | 10200 | 8240 | 256 | 270 | 244 |
| $2.00\times10^{-5}$ | 300 | 1 | Dec. 6 | 117.5 | 23.50 | 0550 | 12500 | 14300 | 11200 | 231 | 242 | 222 |
| | 500 | | Nov. 26 | 126.6 | 26.50 | 0617 | 14000 | 15400 | 12900 | 229 | 237 | 222 |
| $1.25\times10^{-5}$ | 300 ±10 percent | 1 | Nov. 21 | 110.1 | 22.00 | 0531 | 5650 | 6930 | 4700 | 257 | 287 | 228 |

[a]See appendix B for time conversion.




(a) View from North Pole.




(b) Equatorial view.

Figure 1. - Effect of oblateness of Earth on orbit.

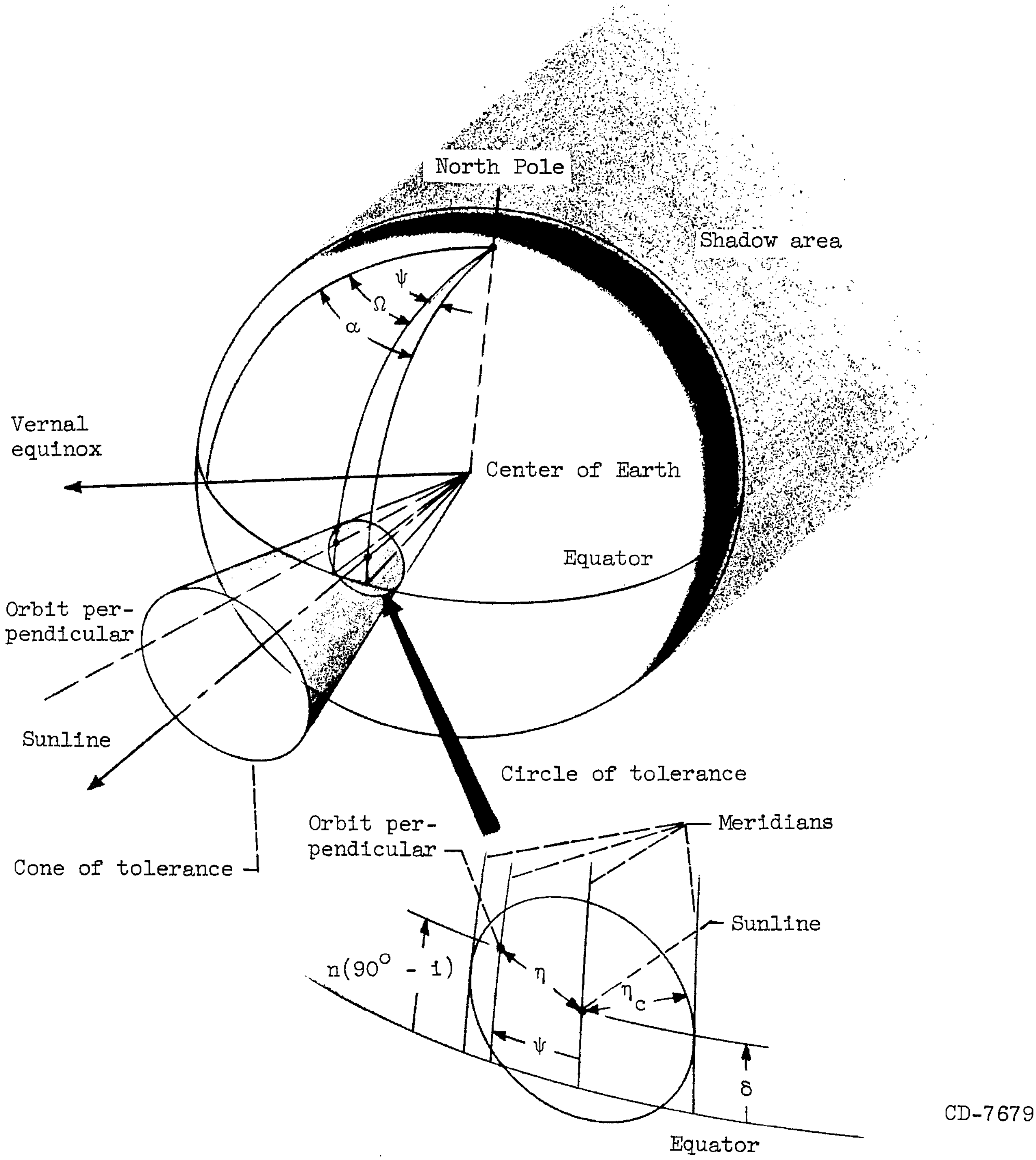



Figure 2. - Cone of tolerance associated with Earth.

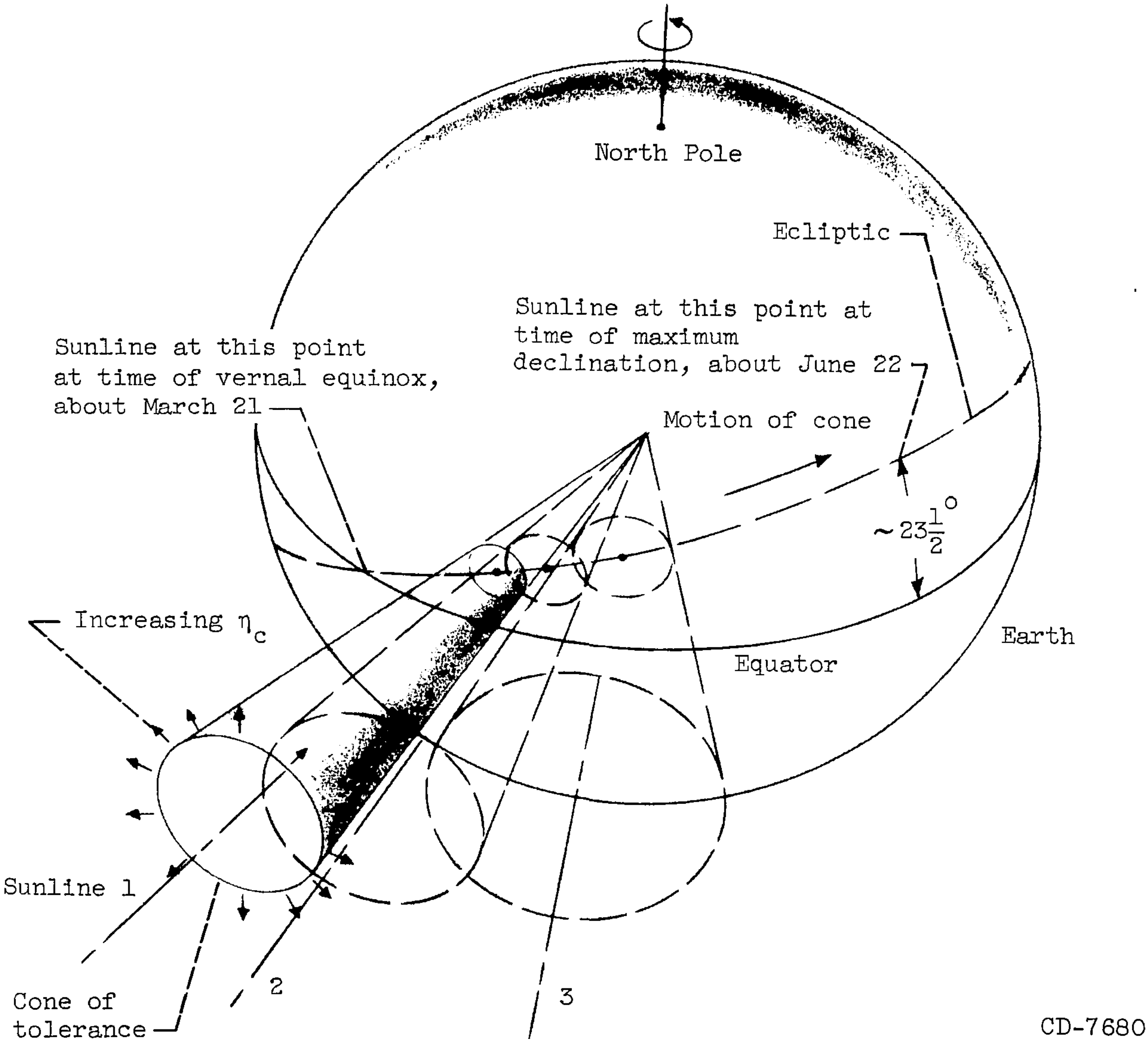



Figure 3. - Cone of tolerance in motion about Earth. Earth rotation does not change geometry.

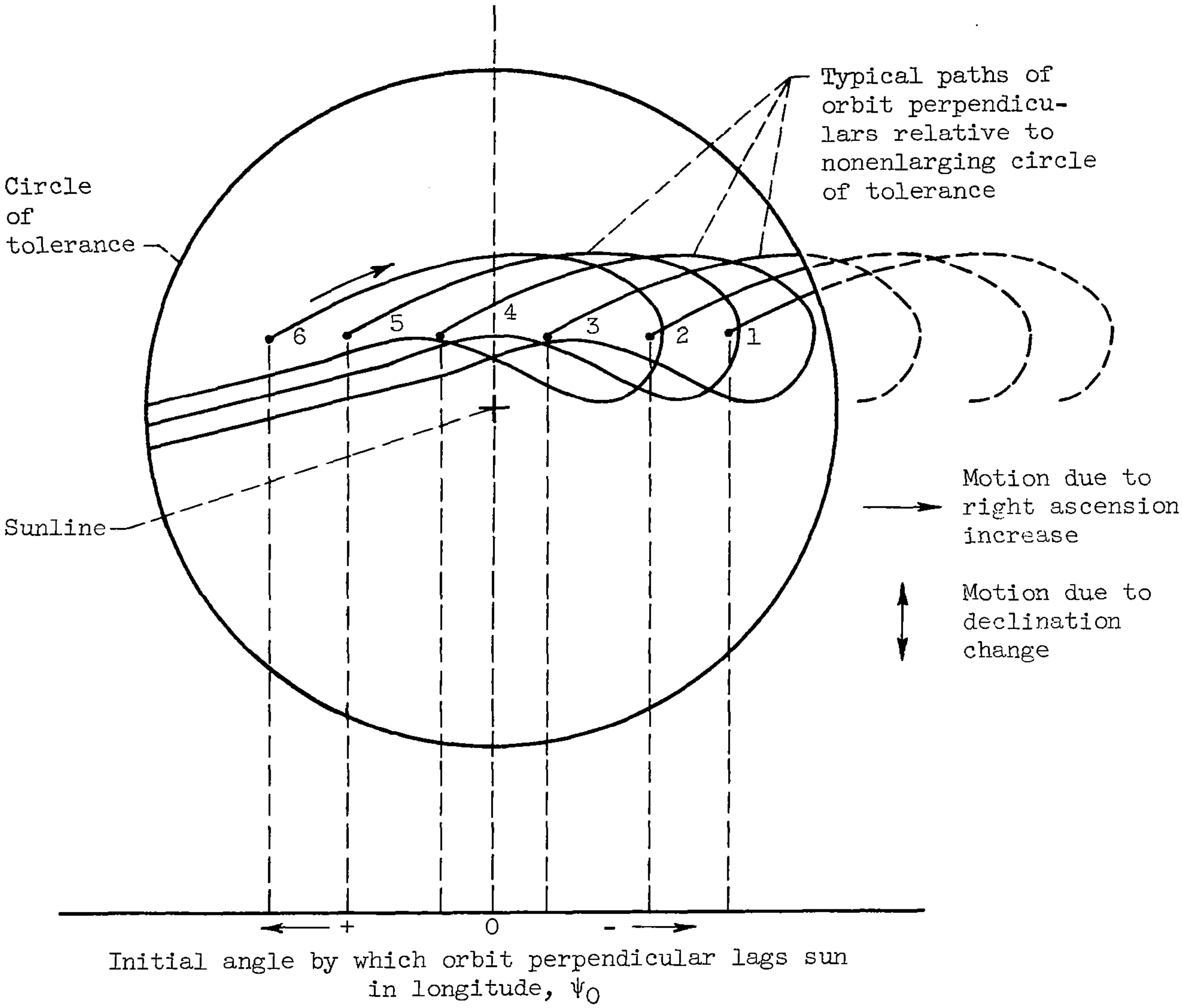



Figure 4. - Typical paths of orbit perpendicular. If mission terminates on left side of sunline, upper altitude curves apply; if mission terminates on right side of sunline, lower altitude curves apply.

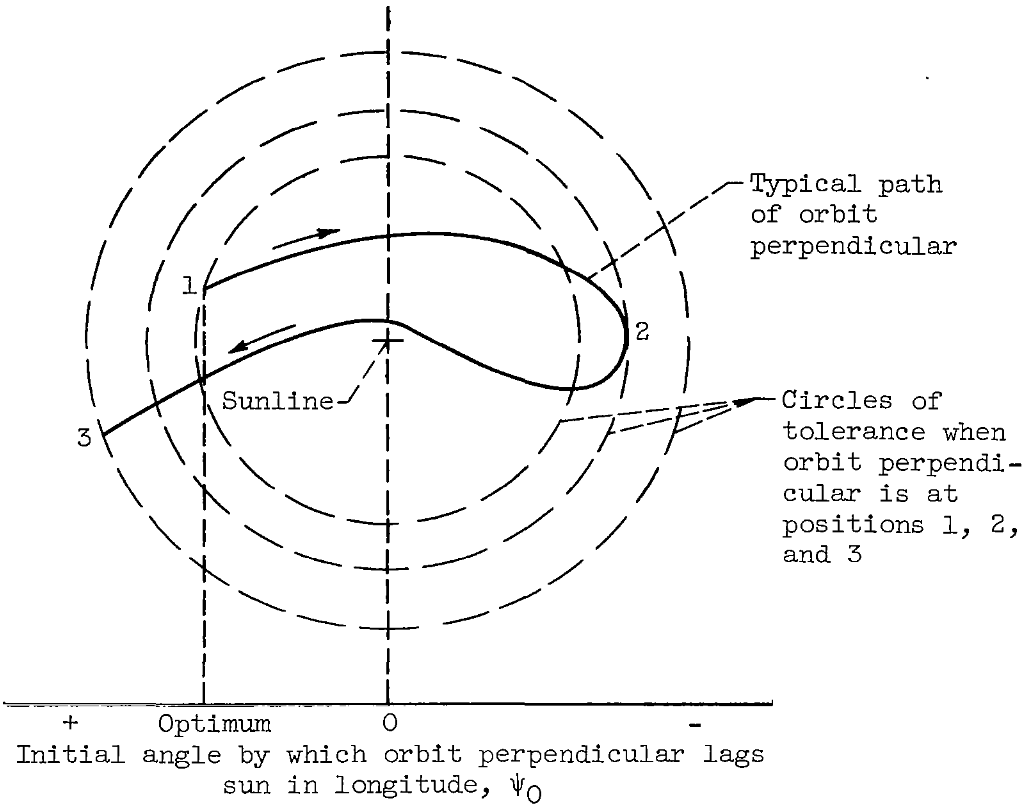



Figure 5. - Path of orbit perpendicular for maximum altitude.

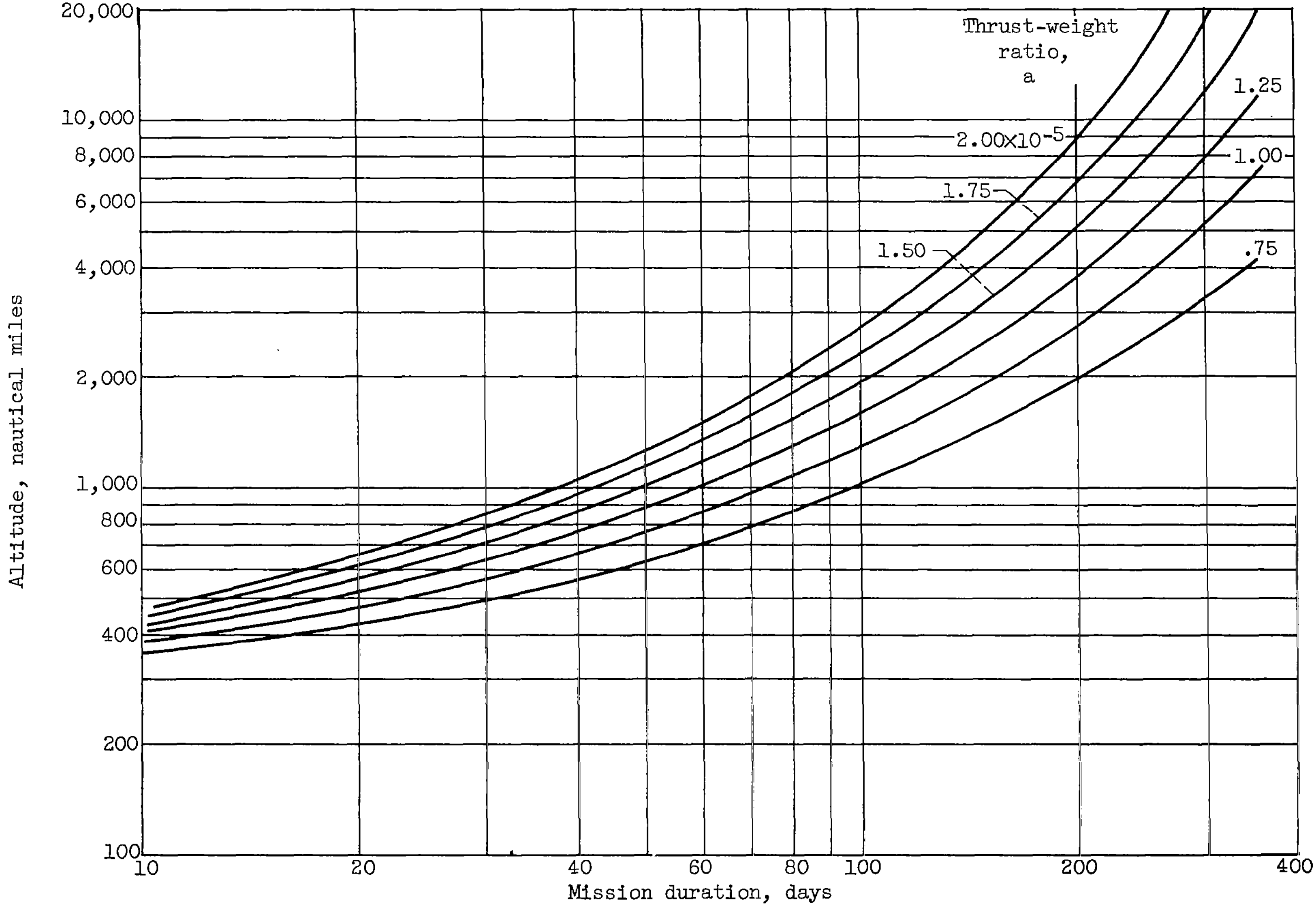



Figure 6. - Altitude as function of thrusting time. Initial altitude, 300 nautical miles.

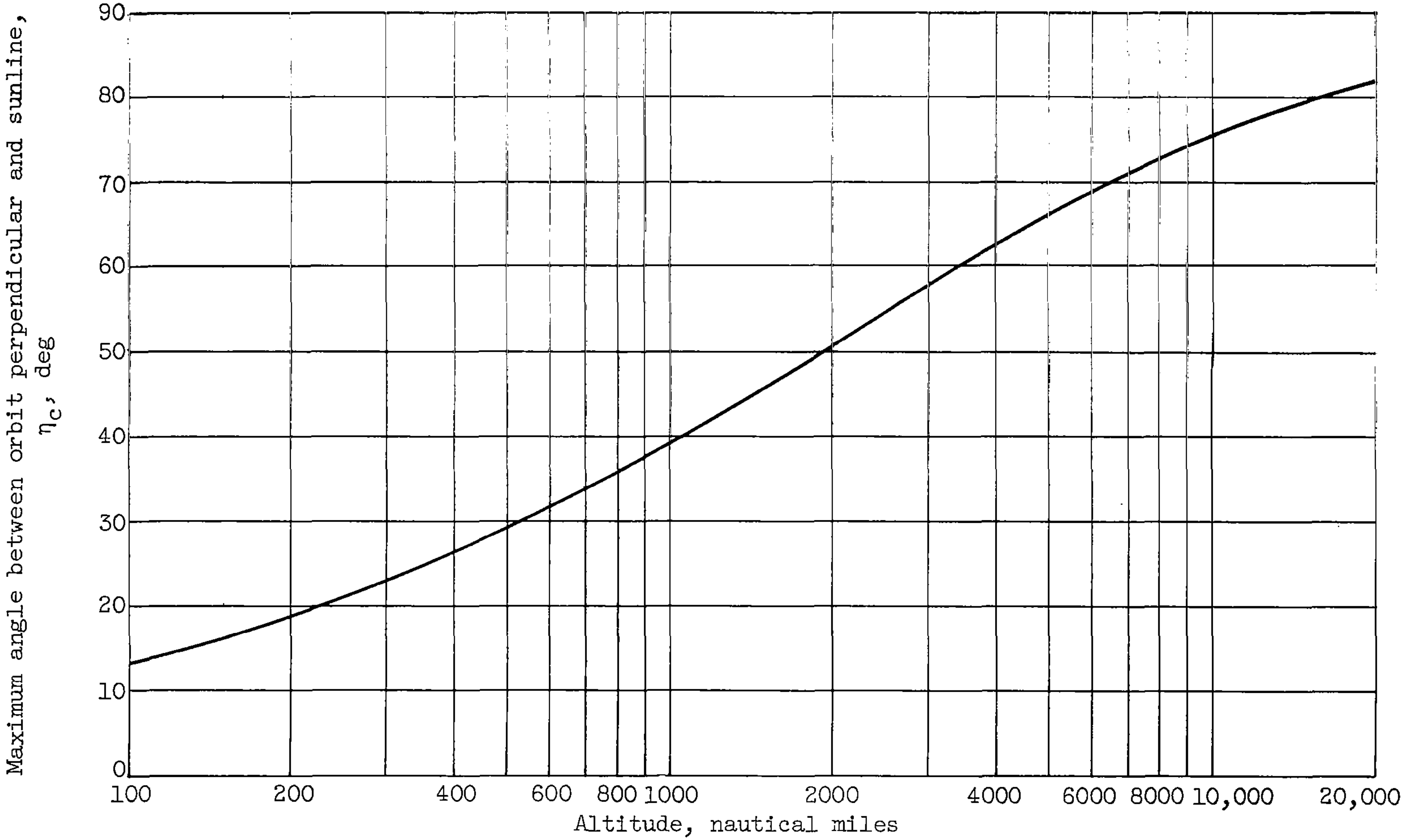
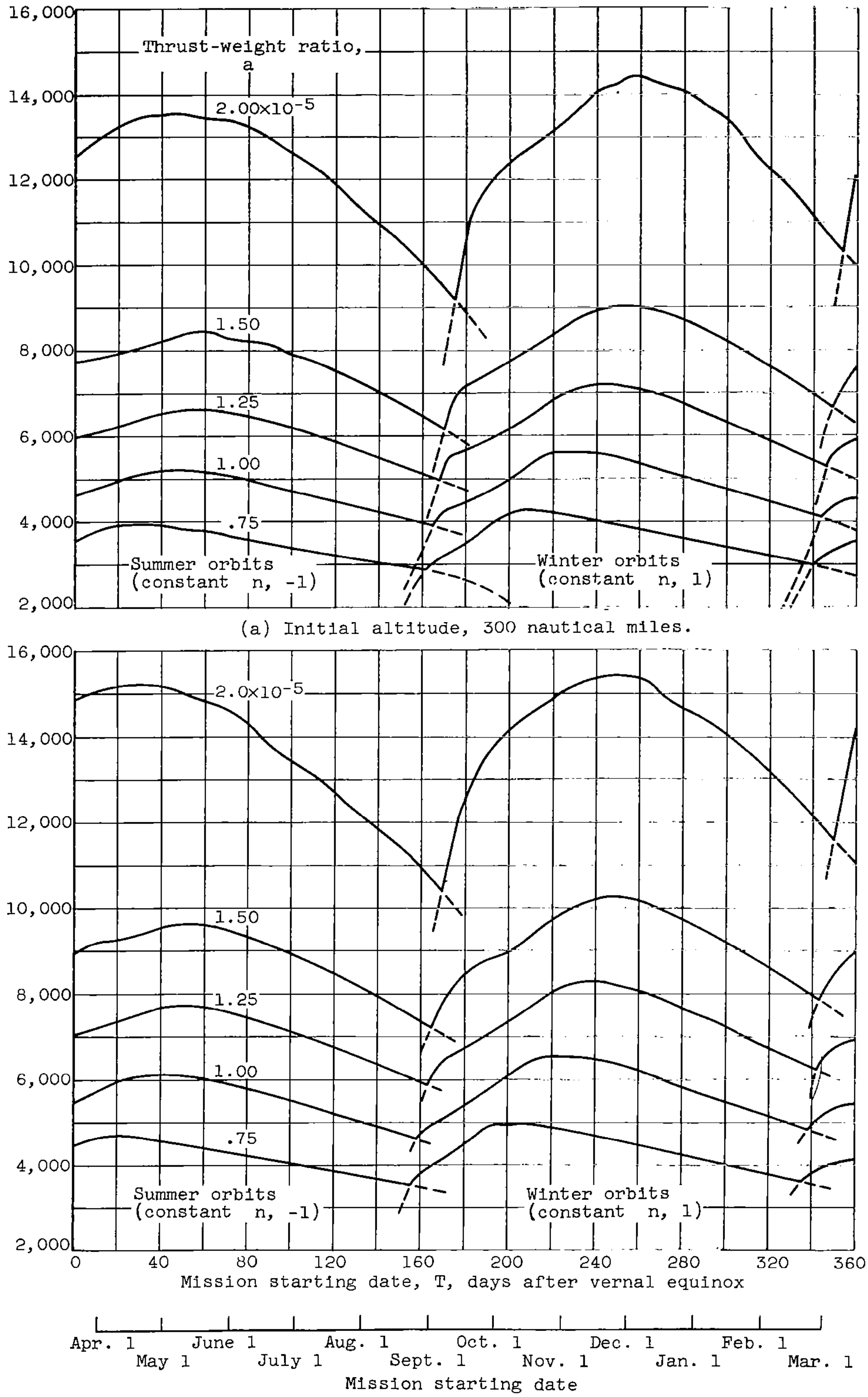



Figure 7. - Maximum angle between orbit perpendicular and sunline as function of altitude.




(b) Initial altitude, 500 nautical miles

Figure 8. - Maximum altitude as function of mission starting date.

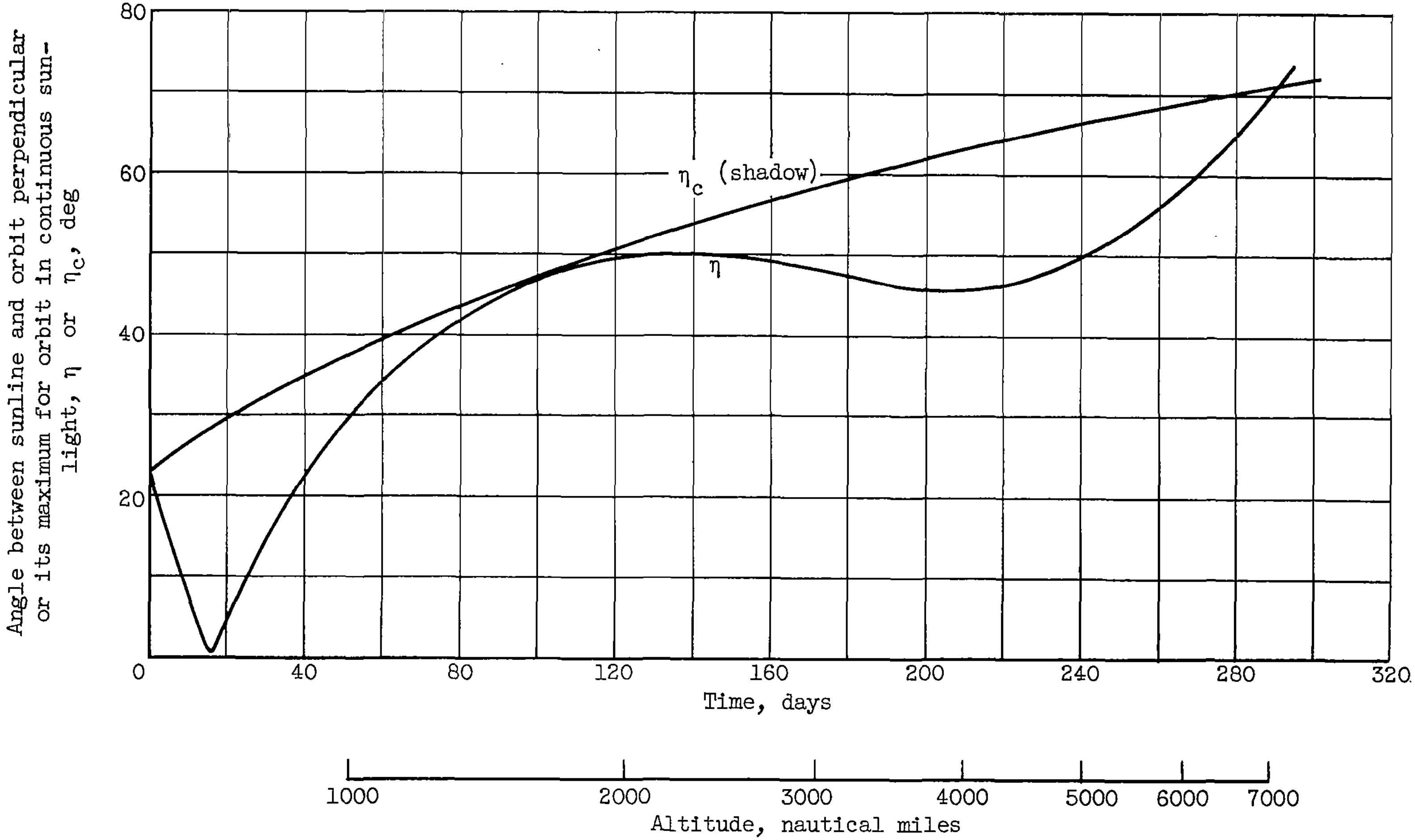
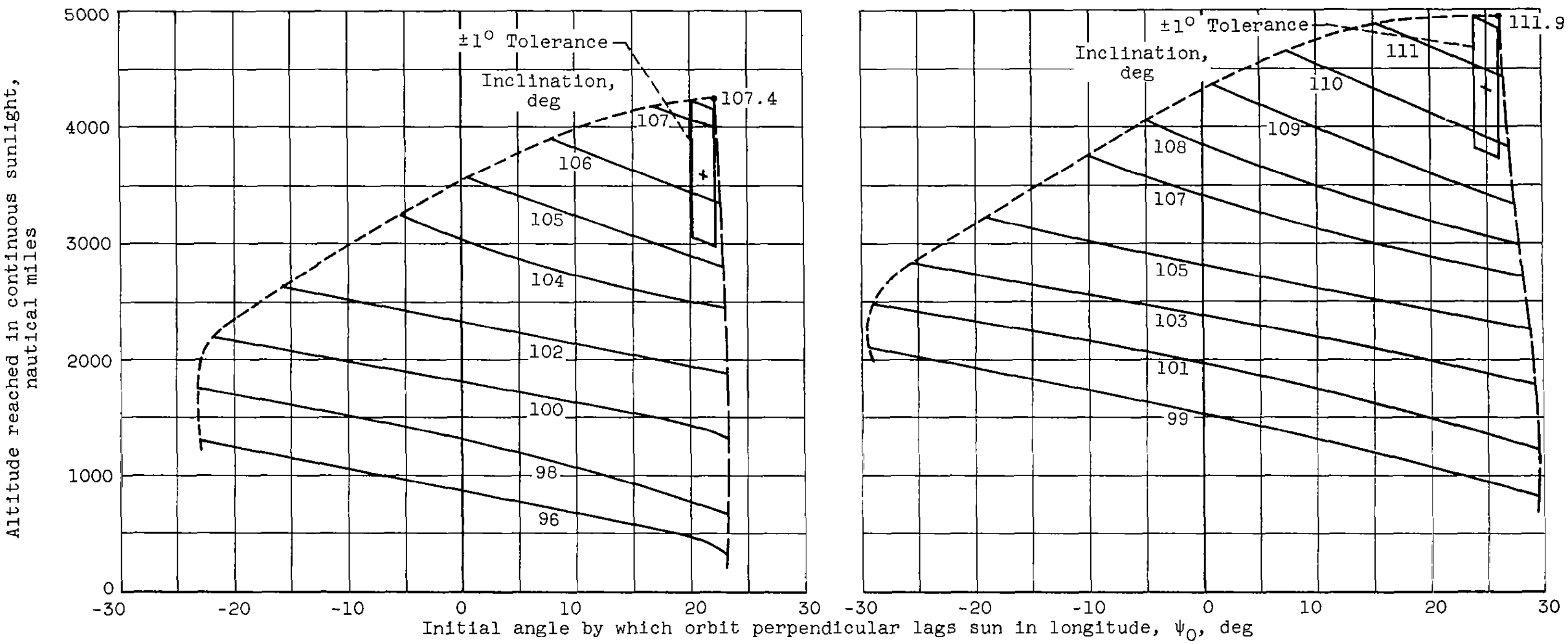



Figure 9. - Angle between sunline and orbit perpendicular and its maximum for orbit in continuous sunlight for typical mission. Thrust-weight ratio, $1.25 \times 10^{-5}$; initial altitude, 300 nautical miles; constant n, 1; mission starting date, November 21; inclination, 112.2°; initial angle by which orbit perpendicular lags sun in longitude, 24.63°; no tolerance on initial orbit.

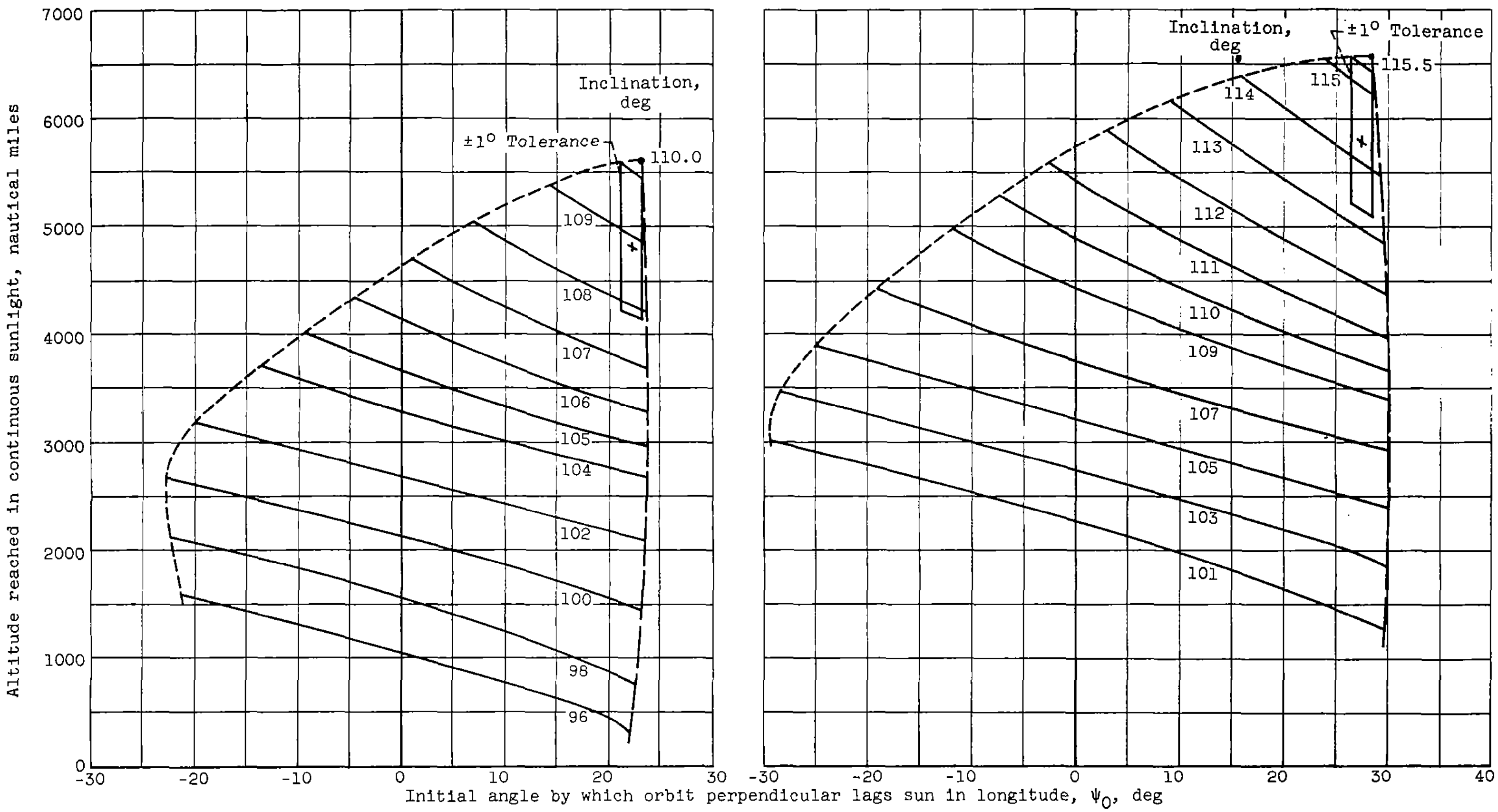



(a-1) Initial altitude, 300 nautical miles; mission starting date, October 17.

(a-2) Initial altitude, 500 nautical miles; mission starting date, October 12.

(a) Thrust-weight ratio, $0.75 \times 10^{-5}$.

Figure 10. - Altitude as function of initial angle by which orbit perpendicular lags sun in longitude. Constant n, 1.

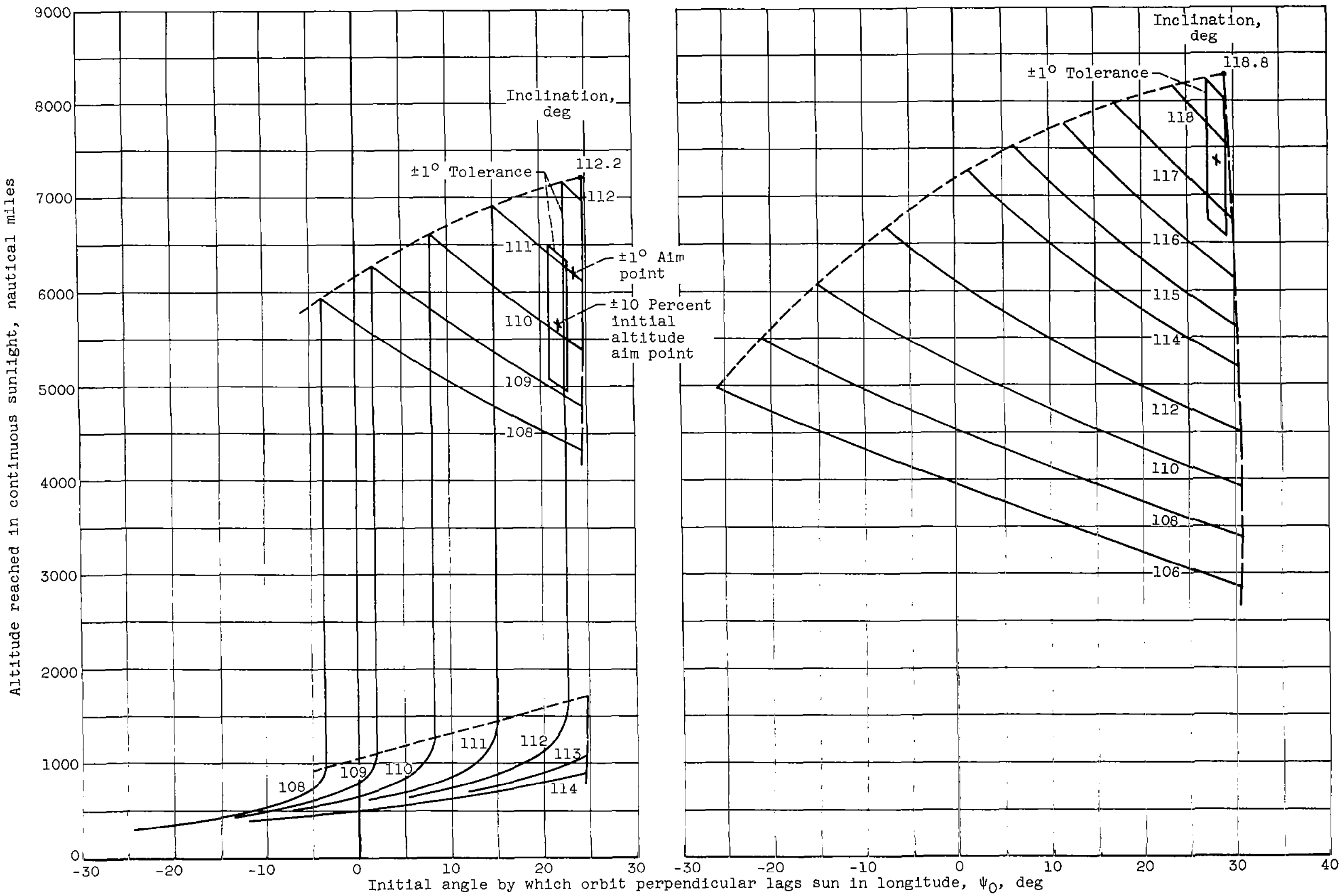



(b-1) Initial altitude, 300 nautical miles.

(b-2) Initial altitude, 500 nautical miles.

(b) Thrust-weight ratio, $1.00 \times 10^{-5}$; mission starting date, November 1.

Figure 10. - Continued. Altitude as function of initial angle by which orbit perpendicular lags sun in longitude. Constant n, 1.

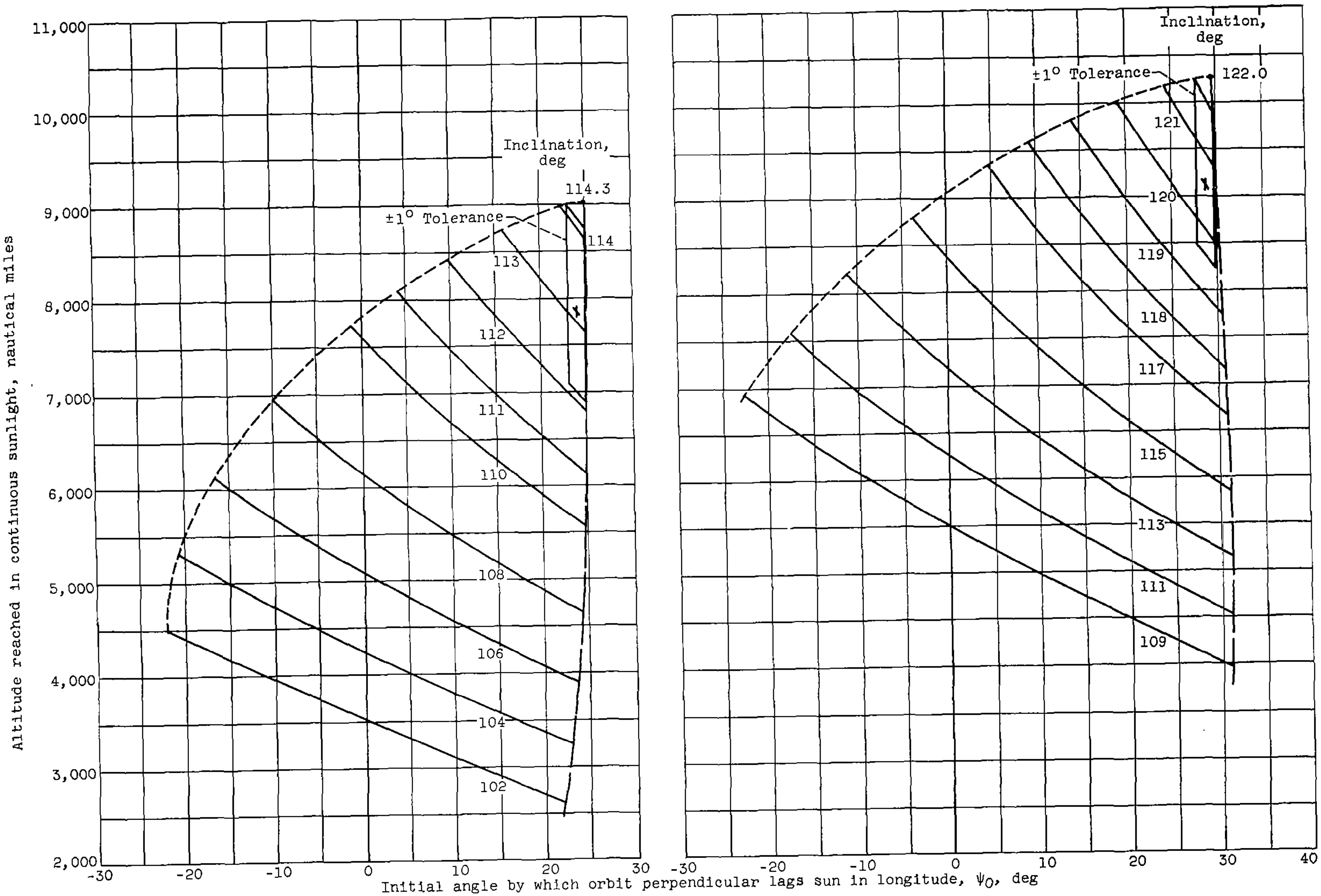



(c-1) Initial altitude, 300 nautical miles; mission starting date, November 21.

(c-2) Initial altitude, 500 nautical miles; mission starting date, November 11.

(c) Thrust-weight ratio, $1.25 \times 10^{-5}$.

Figure 10. - Continued. Altitude as function of initial angle by which orbit perpendicular lags sun in longitude. Constant n, 1.

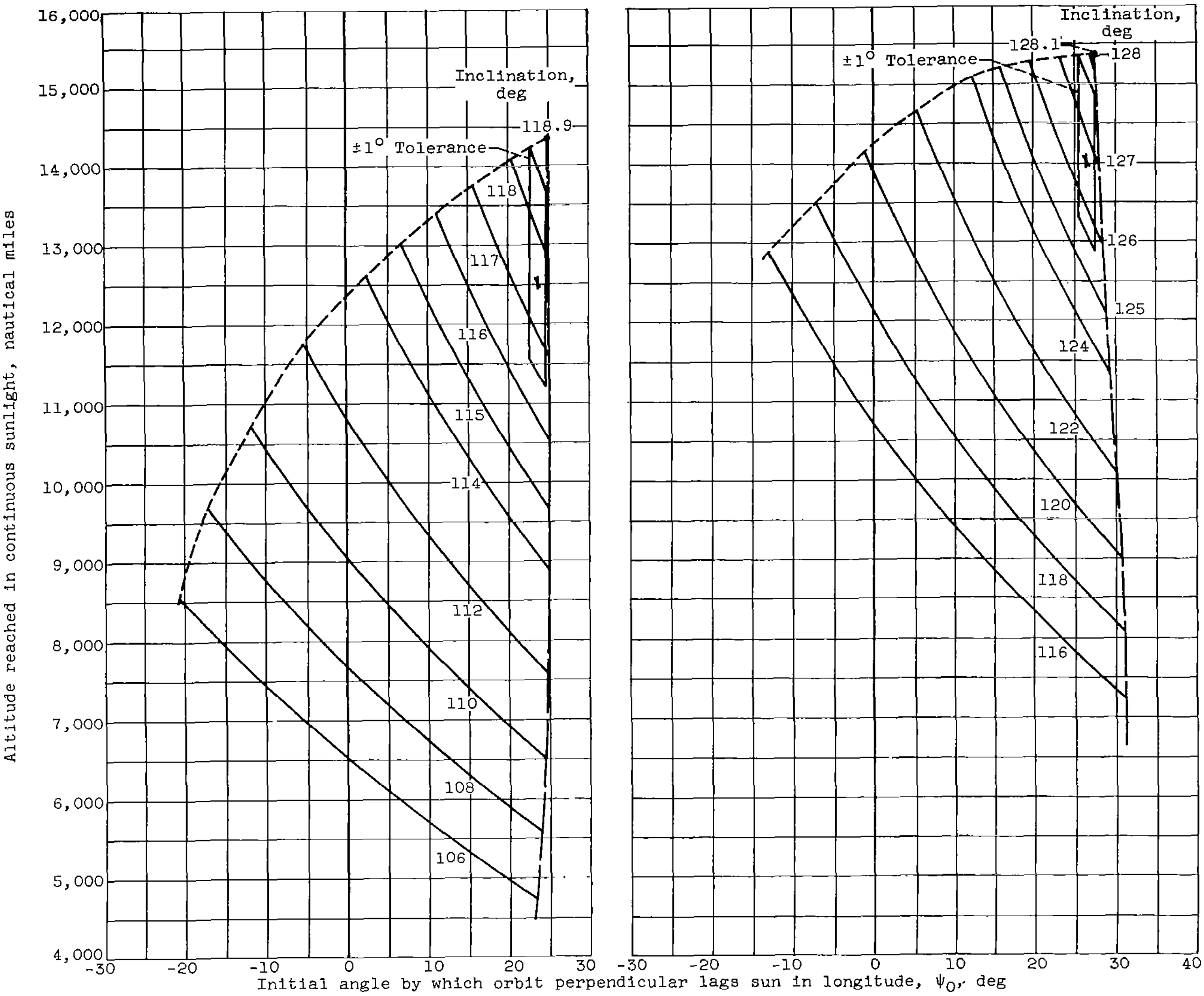



(d-1) Initial altitude, 300 nautical miles; mission starting date, December 1.

(d-2) Initial altitude, 500 nautical miles; mission starting date, November 2.

(d) Thrust-weight ratio, $1.50 \times 10^{-5}$.

Figure 10. - Continued. Altitude as function of initial angle by which orbit perpendicular lags sun in longitude. Constant n, 1.

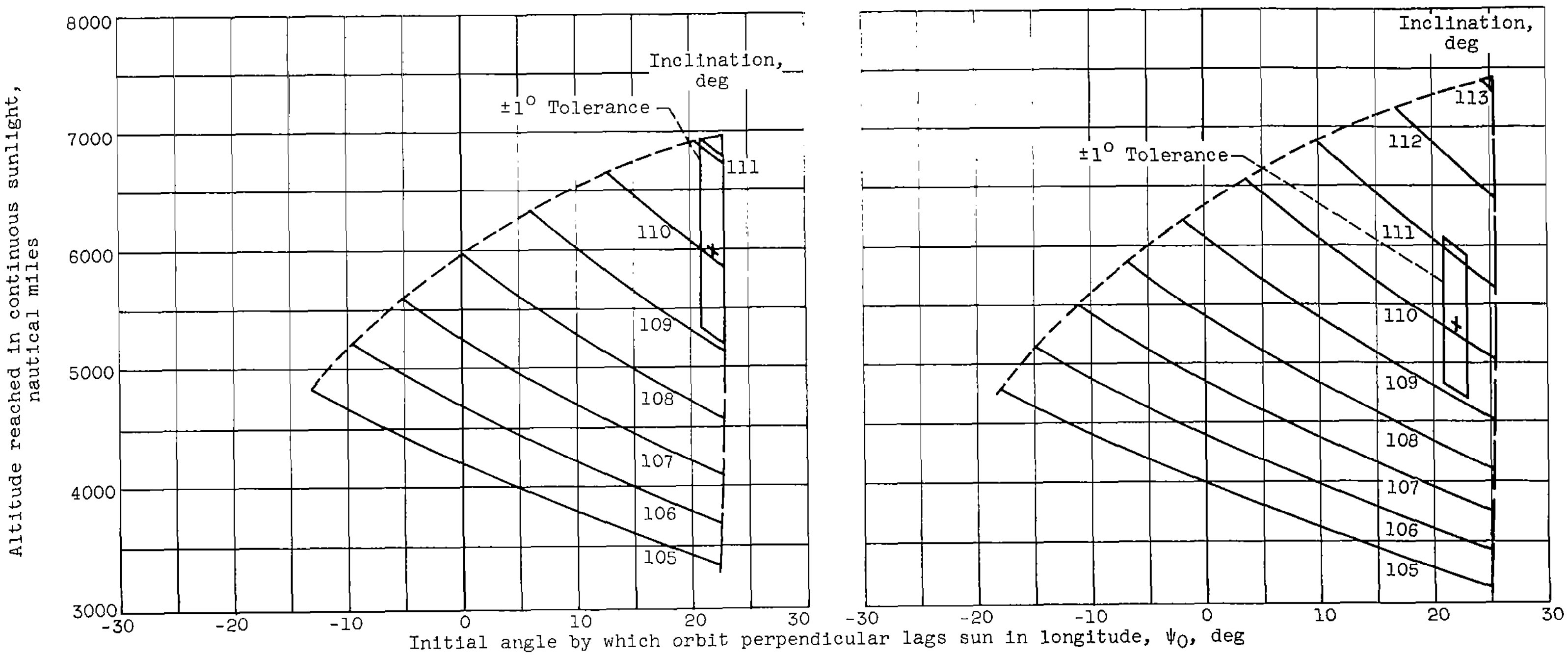



(e-1) Initial altitude, 300 nautical miles; mission starting date, December 6.

(e-2) Initial altitude, 500 nautical miles; mission starting date, November 26.

(e) Thrust-weight ratio, $2.00 \times 10^{-5}$.

Figure 10. - Concluded. Altitude as function of initial angle by which orbit perpendicular lags sun in longitude. Constant n, 1.




(a) Initial altitude, 270 nautical miles.

(b) Initial altitude, 330 nautical miles.

Figure 11. - Altitude as function of initial angle by which orbit perpendicular lags sun in longitude for initial altitudes equal to nominal value plus and minus tolerance. Thrust-weight ratio, $1.25\times10^{-5}$; mission starting date, November 21; constant n, 1.

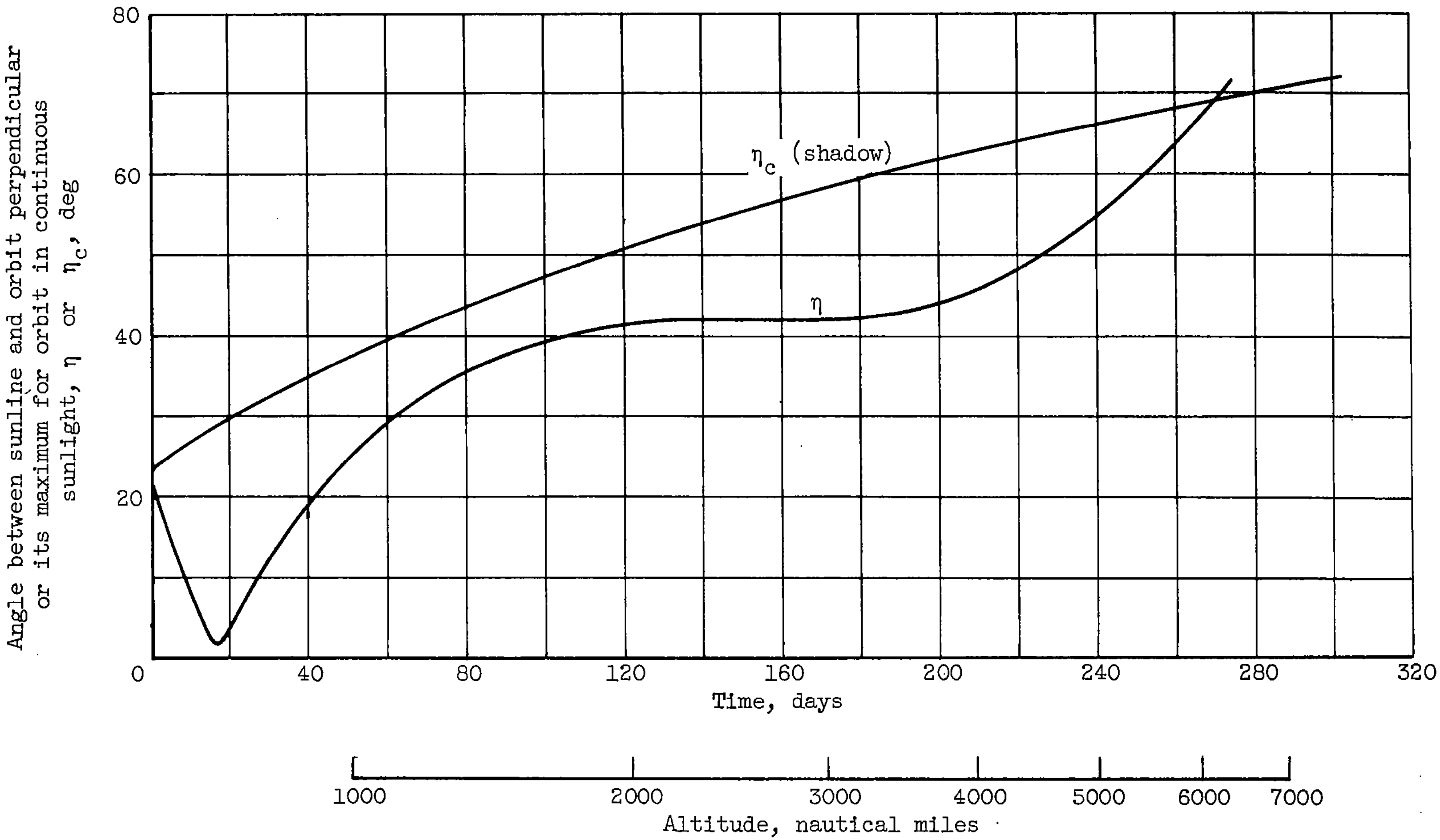



Figure 12. - Angle between sunline and orbit perpendicular and its maximum for orbit in continuous sunlight for two typical missions. Thrust-weight ratio, $1.25 \times 10^{-5}$; initial altitude, 300 nautical miles; mission starting date, November 21; inclination, 111.0° (±1°); initial angle by which orbit perpendicular lags sun in longitude, 23.75° (±1°).

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